Deep Learning

7.1 Transposed Convolutions

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Transposed Convolution

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4. Transposed Convolutions increase the output size

Revisit Convolution in deep learning

1. 1D convolution $x \circledast y$

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\begin{aligned}
y_i &= \sum_a x_{i+a-1} k_a \\
&= \sum_u x_u k_{u-i+1}
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$$

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2. Backpropagating through convolution

$$\left[\frac{\partial l}{\partial x}\right]_u = \left[\frac{\partial l}{\partial x_u}\right]$$
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3. This also looks like convolution, but the kernel is visited in the reverse order

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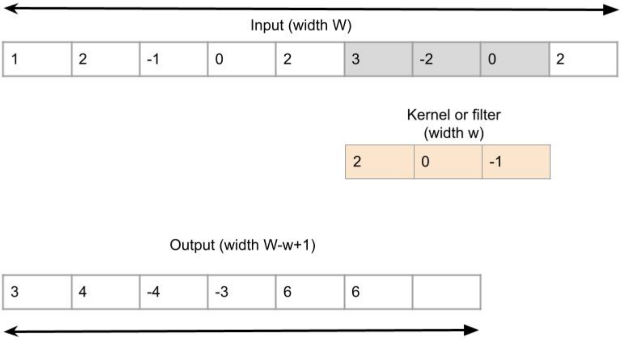
3. Therefore the backprop in convolution becomes

$$\text{if } y = x \circledast k,$$
$$\text{then } \left[\frac{\partial l}{\partial x}\right] = \left[\frac{\partial l}{\partial y}\right] * k$$

1D convolution example



1D Convolution as matrix operation

$$\begin{bmatrix} 2 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 2 & 0 & -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 2 & 0 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 2 & 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 2 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 2 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 2 & 0 & -1 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ -1 \\ 0 \\ 2 \\ 3 \\ -2 \\ 0 \\ 2 \end{bmatrix} = \begin{bmatrix} 3 \\ 4 \\ -4 \\ -3 \\ 6 \\ 6 \\ -6 \end{bmatrix}$$

Backprop as a matrix operation

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- F.conv_transpose1d implements this operation.

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2. Used in Deep generative models