Deep Learning

4.1 Convolution

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CNNs

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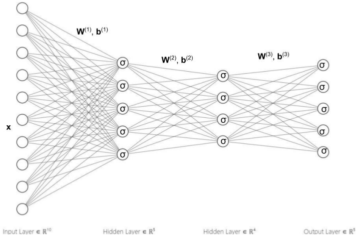
4. So, what changes?

An MLP

1. Input is a vector



An MLP

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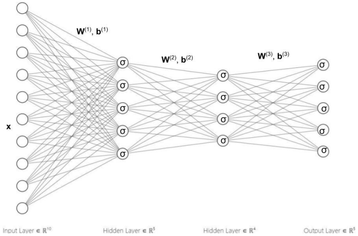
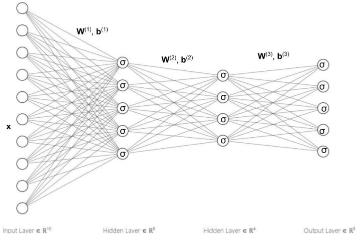
2. Series of densely connected hidden layers



An MLP

1. Input is a vector
2. Series of densely connected hidden layers
3. Neurons in each layer are independent



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5. Flattening removes the structure

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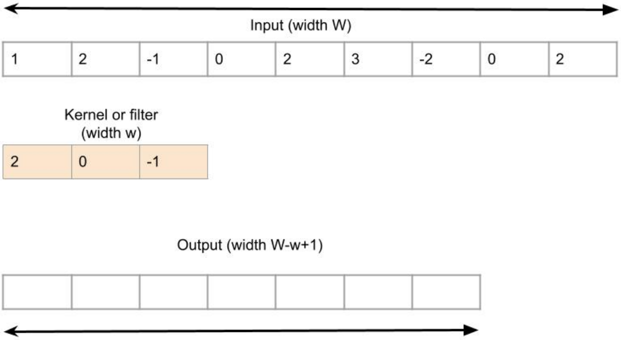
2. Features may occur at different locations in the signal
3. Convolution incorporates this idea: Applies same linear operation at all the locations and preserves the structure

Convolution



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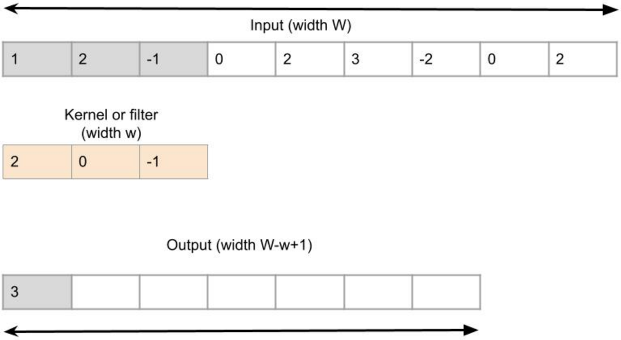

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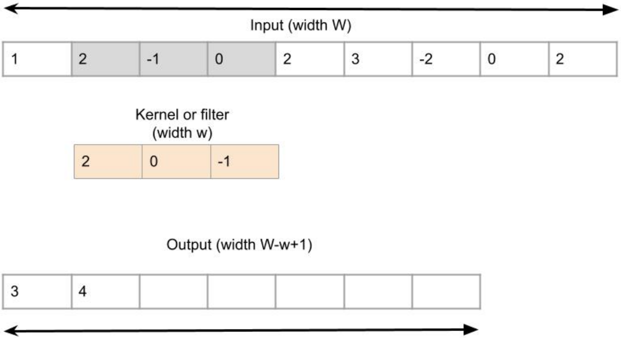

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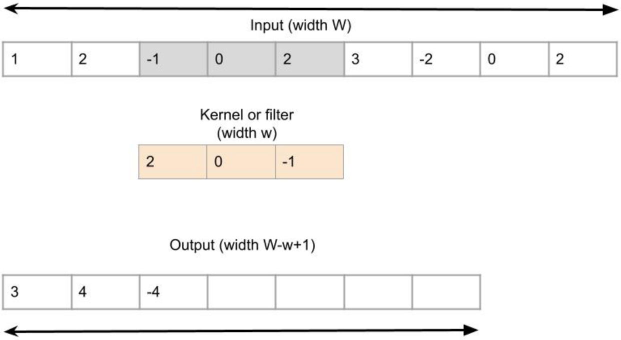

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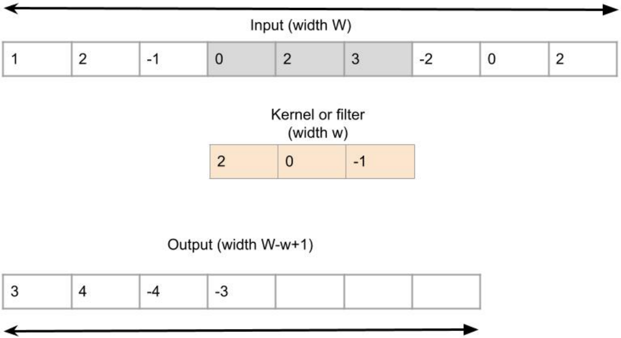

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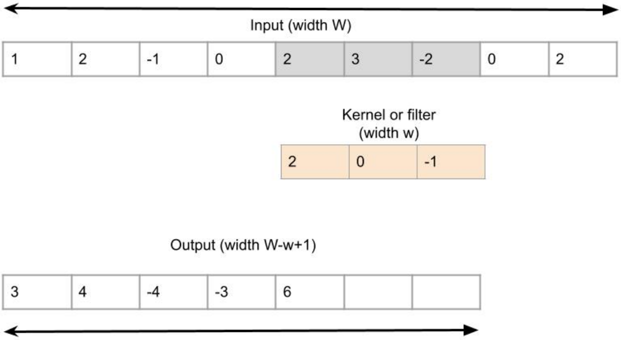

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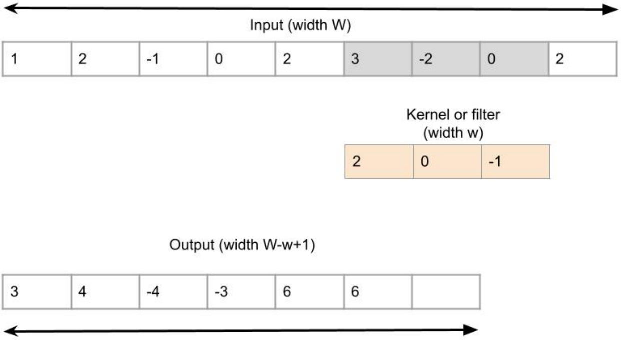

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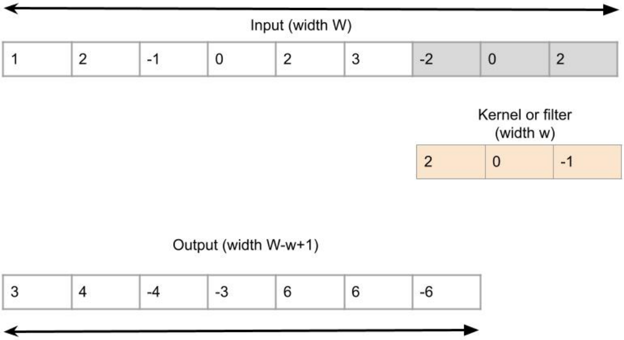

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   - if the i/p is a 2D tensor $\rightarrow$ o/p is also a 2D tensor
   - There exist a relation between the locations of i/p and o/p values

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2. The result $(x \circledast k)$ of convolving $\mathbf{x}$ with $\mathbf{k}$ will be a 1D tensor of size $W - w + 1$

$$
\begin{aligned}
(x \circledast k)_i &= \sum_{j=1}^{w} x_{i-1+j} k_j \\
&= (x_i, \ldots x_{i+w-1}) \cdot \mathbf{k}
\end{aligned}
$$

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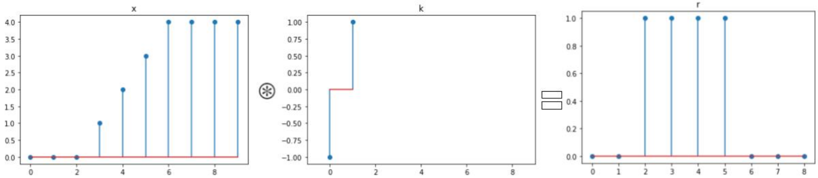
$$(0, 0, 0, 1, 2, 3, 4, 4, 4, 4) \circledast (-1, 1) = (0, 0, 1, 1, 1, 1, 0, 0, 0)$$



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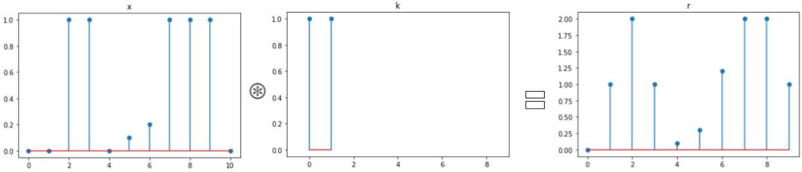
$$(0, 0, 1, 1, 0, 0.1, 0.2, 1, 1, 1, 0) \circledast (1, 1) = (0, 1, 2, 1, 0.1, 0.3, 1.2, 2, 2, 1)$$



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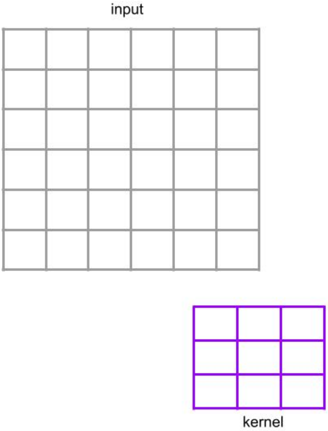
2. In their most usual form, CNNs process 3D tensors of size $C \times H \times W$ with kernels of size $C \times h \times w$ and result in 2D tensors of size $H - h + 1 \times W - w + 1$
3. Note that we generally refer to these inputs as 2D signal (despite having C channels), because, they are referenced as vectors indexed by 2d locations without structure in the channel dimension

2D Convolution



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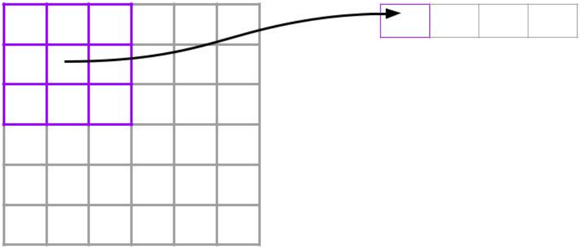

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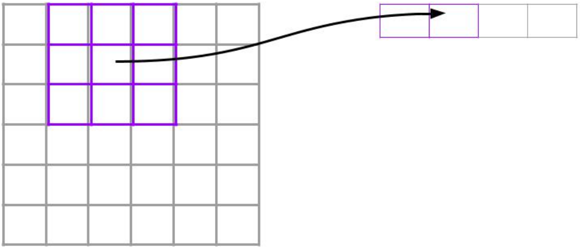

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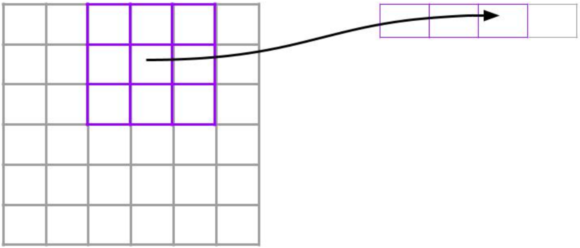

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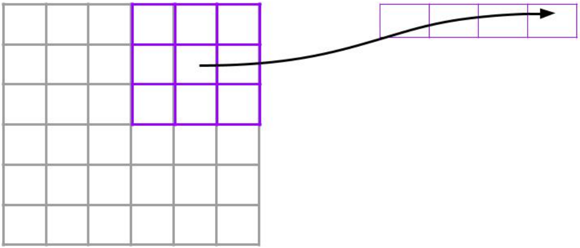

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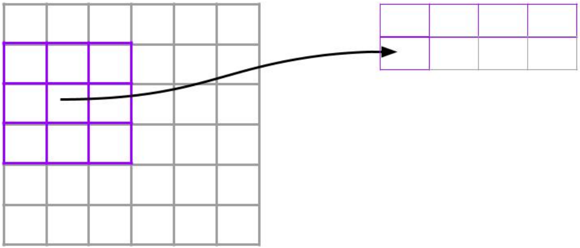

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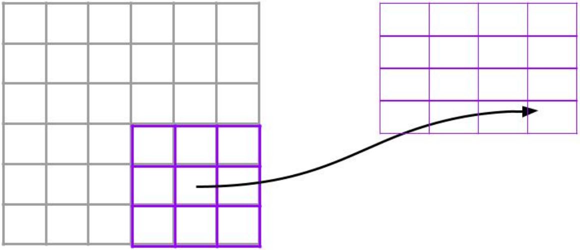

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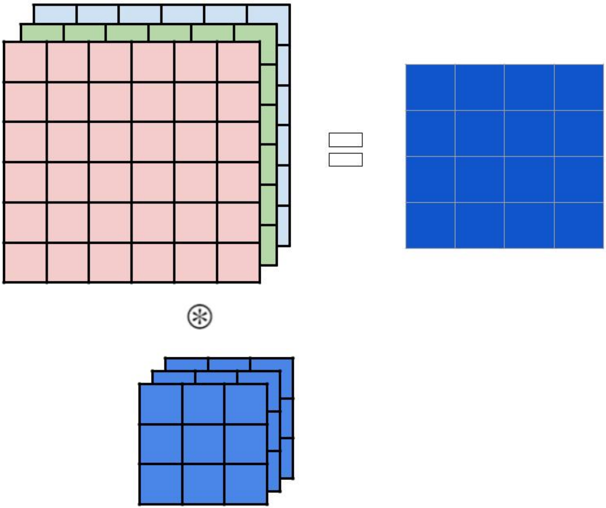

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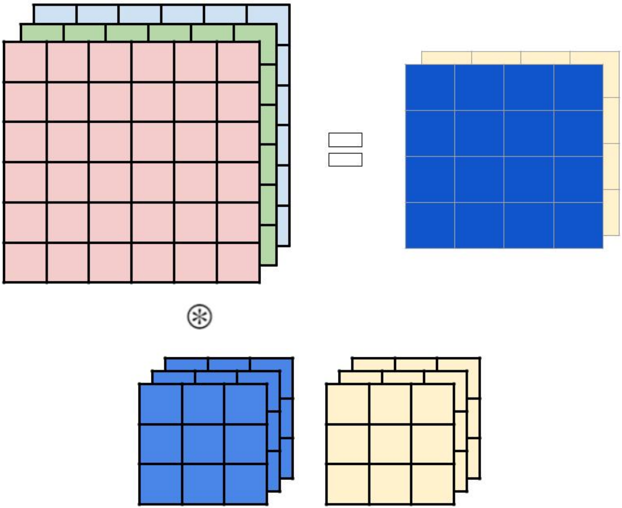

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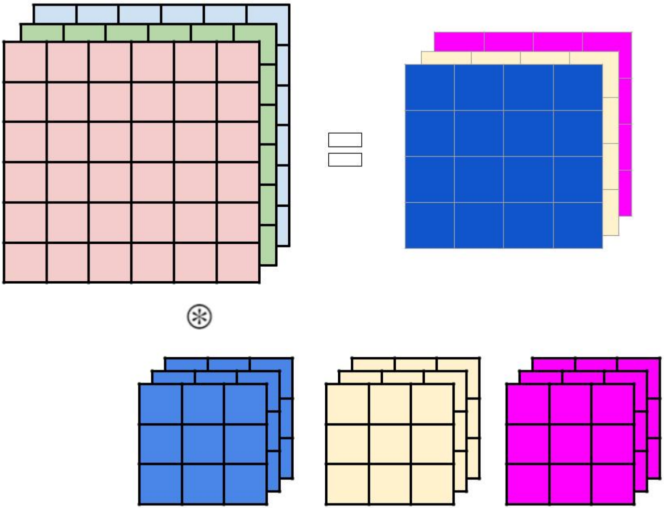

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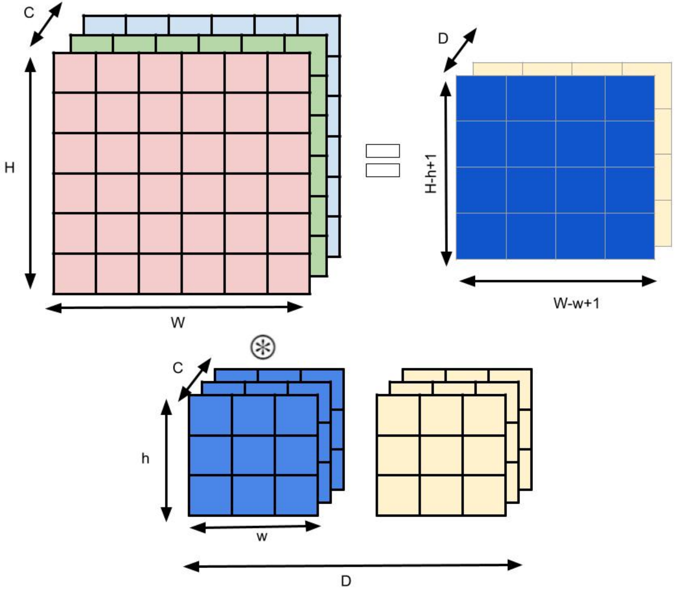

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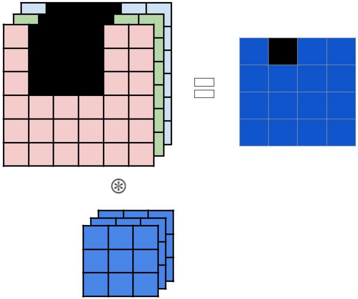

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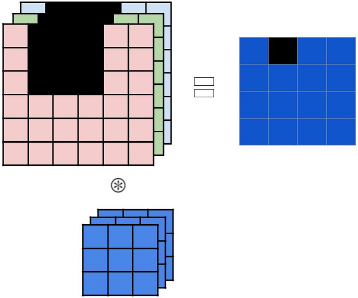

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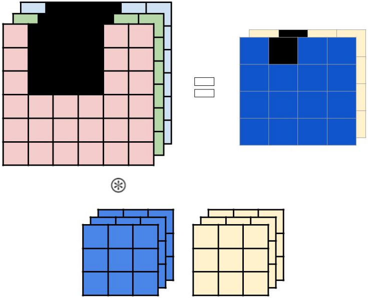

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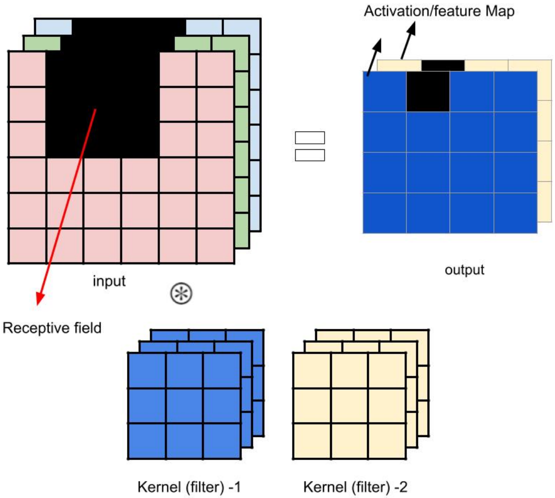
   - Adjacent components in o/p are influenced by adjacent parts in the i/p
2. If the channel dimension has a metric meaning (e.g. time) 3D convolution can be employed (e.g. frames in a video)

Terminology in Convolution



Convolution function in PyTorch

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4. `input` is $N \times C \times H \times W$ dimensional signal
5. Output is $N \times D \times (H - h + 1) \times (W - w + 1)$ tensor

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5. Output is $N \times D \times (H - h + 1) \times (W - w + 1)$ tensor
6. Autograd compliant

Convolution function in PyTorch

```
input = torch.empty(128, 3, 20, 20).normal_()
weight = torch.empty(5, 3, 5, 5).normal_()
bias = torch.empty(5).normal_()
output = F.conv2d(input, weight, bias)
output.size()
torch.Size([128, 5, 16, 16])
```

Look/Access the filters

```
weight[0,0]
tensor([[-0.6974, 0.1342, -0.2632, -0.4672, 0.1827],
[-0.1184, -0.2164, 0.2772, -0.1099, 0.0103],
[-0.8272, 0.3580, 0.2398, -0.5795,-0.9472],
[-1.1734, -0.1019, 0.7394, 0.3342, 0.1699],
[ 1.9271, 0.1250, 0.4222, 0.2014, 1.1100]])
```

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3. Encloses the convolution as a module
4. Initializes the kernel parameters and biases as random

Conv layer in PyTorch

```
f = nn.Conv2d(in_channels = 3, out_channels = 5,
kernel_size = (2, 3))
for n, p in f.named_parameters():
...print(n, p.size())

>>weight torch.Size([5, 3, 2, 3])
>>bias torch.Size([5])
```

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kernel_size = (2, 3))
for n, p in f.named_parameters():
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>>weight torch.Size([5, 3, 2, 3])
>>bias torch.Size([5])

input = torch.empty(128, 3, 28, 28).normal_()
output = f(input)
output.size()
>>torch.Size([128, 5, 27, 26])
```

Padding in Convolution

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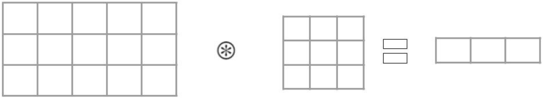
3. Instead of zeros, one may pad with signal values at the edges

Padding in Convolution



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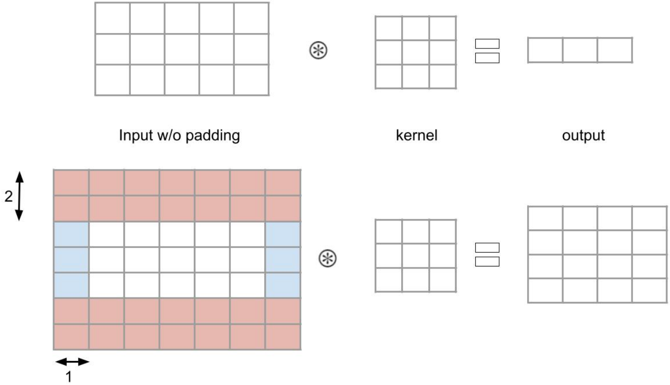

Stride in Convolution

1. Specifies the step size taken while performing convolution

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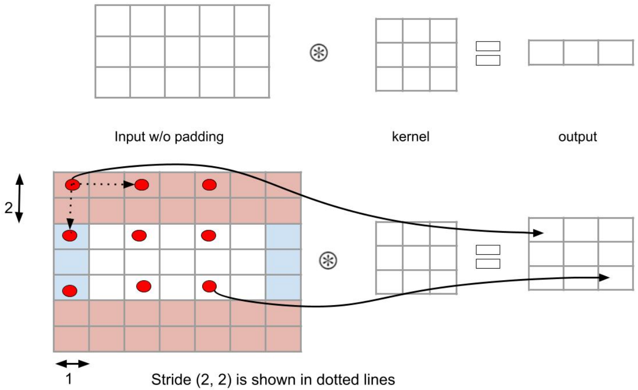
2. Default value is 1, i.e., move the kernel across the signal densely (without skipping)

Padding and Stride in Convolution



Dilation in Convolution

1. Manipulates the size of the kernel via expanding its size without adding weights.

Dilation in Convolution

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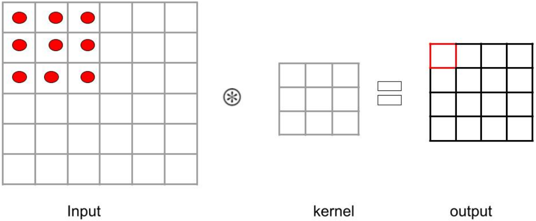
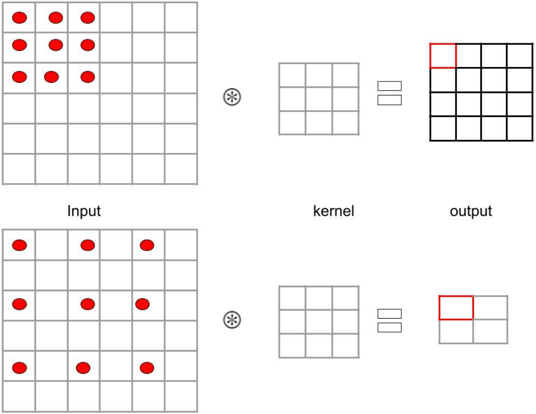
2. In other words, it inserts 0s in between the kernel values

Without Dilation



Dilation (2, 2)



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4. Dilation increases the receptive field
5. It is referred to as 'atrous' convolution