भारतीय प्रौद्योगिकी संस्थान तिरुपति
IIT TIRUPATI

Deep Learning

3.5 More on Gradient Descent

Dr. Konda Reddy Mopuri
kmopuri@iittp.ac.in
Dept. of CSE, IIT Tirupati

Gradient Descent

1. By far the most common way to train neural networks
2. DL libraries provide various ways of implementing Gradient Descent

Recap the Loss in Logistic Regression

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3. This is vanilla or Batch Gradient Descent
4. Sometimes very slow and intractable (datasets that do not fit in the memory)
5. It doesn't allow updating the model online (i.e., with the arrival of new data samples, on the fly)

Batch Gradient Descent

```
for i in range(nb_epochs):
  params_grad = evaluategradient(lossfunction, data, params)
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1. Batch GS is guaranteed to converge to global minima in case of convex functions, and to a local minima in case of non-convex functions

Stochastic Gradient Descent (SGD)

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   $w = w - \eta \nabla_w \mathcal{L}(w, x^i, y^i)$
2. In case of large datasets, Batch GD computes redundant gradients for similar examples for each parameter update
3. SGD does away with redundancy and generally faster and can be used to learn online

Stochastic Gradient Descent (SGD)

1. However, frequent updates with a high variance cause the objective function to fluctuate heavily

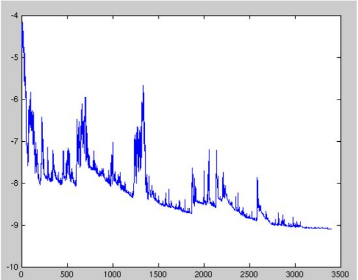

Figure credits: Wikipedia

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1. SGD's fluctuations enable it to jump to new and potentially better local minima
2. This complicates the convergence, as it overshoots
3. However, if the learning rate is slowly decreased, we can show similar convergence to Batch GD

Stochastic Gradient Descent (SGD)

```
for i in range(nb_epochs):
  np.random.shuffle(data)
  for example in data:
    params_grad = evaluate_gradient(loss_function,
example, params)
    params = params - learning_rate * params_grad
```

Mini-batch Gradient Descent

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3. Common mini-batch sizes vary from 50 to 1024, depending on the application
4. This is the algorithm of choice while training DNNs (also, incorrectly referred to as SGD in general)

Mini-batch Gradient Descent

```
for i in range(nb_epochs):
  np.random.shuffle(data)
  for batch in get_batches(data, batch_size = 50):
    params_grad = evaluate_gradient(loss_function, batch,
params)
    params = params - learning_rate * params_grad
```

Some challenges

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1. Choosing a proper learning rate
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   - However, these schedules are defined in advance and hence unable to adapt to the task at hand
2. Same learning rate applies to all the parameters
3. Avoiding numerous sub-optimal local minima

Different update versions in GD

To deal with the discussed challenges, researchers proposed variety of update equations for GD

- SGD with momentum
- Nesterov Accelerated Gradient
- AdaGrad
- Adadelta
- Adam
- RMSProp
- etc.

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2. SGD progresses slowly; oscillating in the ravine

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3. γ is usually set to 0.9

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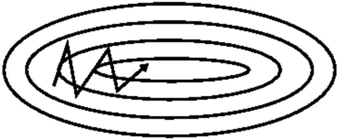
1. Momentum term
   - Increases the update for the components whose gradient points in the same direction
   - Decreases for the dimensions whose gradient change direction across iterations



References

https://ruder.io/optimizing-gradient-descent/