Deep Learning

3.1 Perceptron

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Threshold Logic Unit

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3. Boolean inputs and output

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3. No learning; heuristics approach

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5. $$f(x) = \begin{cases} 1 & \text{when } \sum_i w_i x_i + b \geq 0 \\ 0 & \text{else} \end{cases}$$

Perceptron

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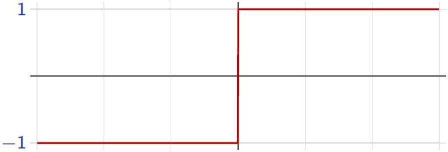

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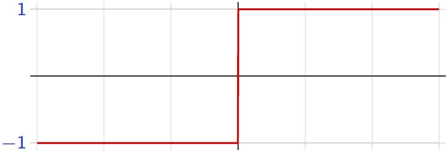

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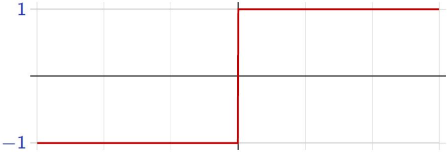

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3. $\mathbf{w}$ are referred to as weights and b as the bias

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4. Mechanism for learning weights

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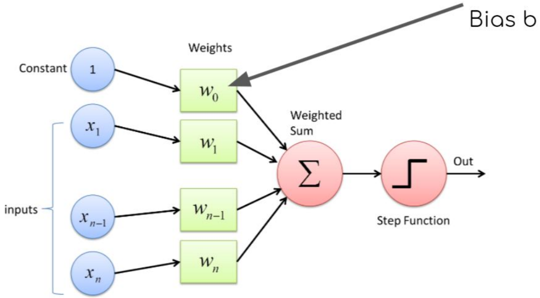
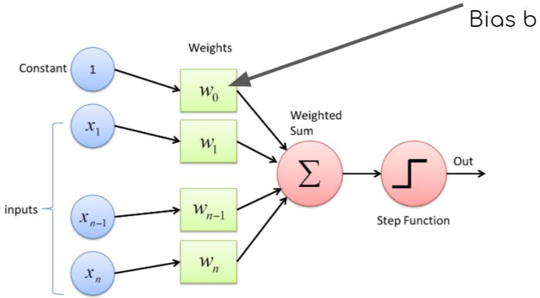


Figure credits: DeepAI

Weights and Bias

1. Why are the weights important?
2. Why is it called ‘bias’? What does it capture?



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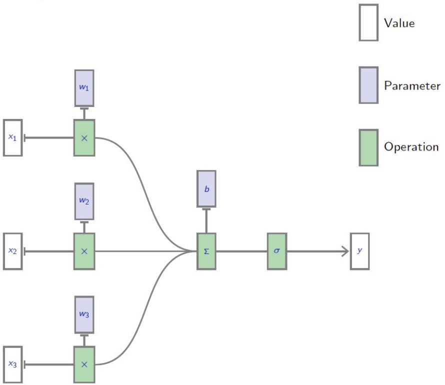

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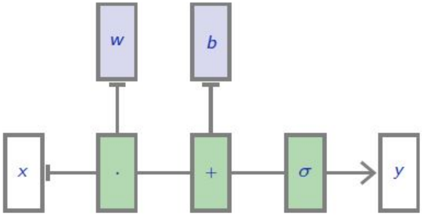

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4. Note that the bias b is absorbed as a component of $\mathbf{w}$ and $\mathbf{x}$ is appended with 1 suitably

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▸ Colab Notebook: Perceptron

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3. Other algorithms maximize the margin from boundary to the samples